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Key Points:

- We test six published iceberg calving models against observations from 50 diverse, marine-terminating glaciers in Greenland
- Position-predicting calving models outperform rate-predicting models when tested against time series observations of calving conditions
- The crevasse depth calving model reproduces calving conditions across space and through time with the highest accuracy

Supporting Information:

Supporting Information S1

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Evaluation of Iceberg Calving Models Against Observations From Greenland Outlet Glaciers

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Abstract Frontal ablation processes at marine-terminating glaciers are challenging to observe and difficult to represent in numerical ice flow models, yet play critical roles in modulating ice sheet mass balance. Current ice sheet models typically rely on simple iceberg calving models to prescribe either terminus positions or iceberg calving rates, but the relative accuracies and uncertainties of these calving models remain largely unconstrained at the ice sheet scale. Here, we evaluate six published iceberg calving models against spatially and temporally diverse observations from 50 marine-terminating outlet glaciers in Greenland. We seek the single model that best reproduces observed conditions across all glaciers, at all observation times, and with low sensitivity to calibration uncertainty. Five of six calving models can produce unbiased estimates of calving position or calving rate at the ice sheet scale. However, time series analysis reveals that, when using a single, optimized model parameter, rate-predicting calving models frequently yield calving rate errors in excess of 10 m d⁻¹. In comparison, terminus position-predicting calving models more accurately track observed changes in terminus position (remaining within ~1 km of the observed grounded terminus position). Overall, our results indicate that the crevasse depth calving model provides the best balance of high accuracy and low sensitivity to imperfect parameter calibration. While the crevasse depth model appears unlikely to capture the true controls on crevasse penetration, numerically, it reproduces observed terminus dynamics with high fidelity and should be considered a leading candidate for use in models of the Greenland Ice Sheet.

1. Introduction

Dynamic ice discharge from marine-terminating outlet glaciers in Greenland has increased substantially over the past two decades and is projected to increase further in coming decades (Aschwanden et al., 2019; Morlighem et al., 2017, 2019; Mouginot et al., 2019). Changes in dynamic discharge are hypothesized to arise at the ice/ocean boundary, where iceberg calving and submarine melting processes (collectively termed frontal ablation) initiate abrupt outlet glacier retreat, triggering accelerated ice flow and upstream thinning (Murray et al., 2010; Nick et al., 2009; Rignot et al., 2010). Realistic simulation of frontal ablation processes at marine-terminating outlet glaciers is therefore critical for accurate forecasts of Greenland Ice Sheet mass change (Vieli & Nick, 2011), yet the physical complexity and observed variability in iceberg calving and submarine melting complicates reliable representation of frontal ablation processes in numerical glacier models (e.g., Bassis & Jacobs, 2013; Benn et al., 2017).

Iceberg calving, the mechanical removal of ice from a glacier terminus, ultimately results from fracture when stresses at the glacier front exceed ice strength. Relevant stresses include longitudinal stresses associated with ice flow, torque and shear stresses arising from buoyancy forces and melt undercutting of the calving front, backpressure from ice mélange, and hydrostatic force imbalance at the ice cliff (Bassis & Jacobs, 2013; Benn, Warren & Mottram, 2007; Van der Veen, 1996). The other frontal ablation process, submarine melting, is a result of heat transfer from ocean waters to the submerged terminus. This process is largely controlled by subglacial runoff flux, ocean water temperatures and salinity, and terminus morphology (Jenkins, 2011; Truffer & Motyka, 2016). Observations from Greenland outlet glaciers suggest that the relative contributions of iceberg calving and submarine melting and their absolute rates are highly variable from glacier to glacier (Enderlin et al., 2013; Rignot, Xu, et al., 2016; Wood et al., 2018), though ~2–50 m d⁻¹ calving losses generally exceed ~1–3 m d⁻¹ submarine melt rates and thus constitute the majority of frontal ablation mass losses (e.g., Joughin et al., 2020; Rignot et al., 2010; Rignot, Xu, et al., 2016; Wagner et al., 2019). Furthermore, submarine melting and iceberg calving are not necessarily independent processes, as

undercutting by melting may destabilize the terminus and promote calving (Bartholomaus et al., 2013; Fried et al., 2015; Luckman et al., 2015). At this time, however, the relationship between calving and melt undercutting remains largely unconstrained and the appropriateness of existing frontal ablation parameterizations is uncertain (Benn et al., 2017; Cook et al., 2014; Ma & Bassis, 2019; O'Leary & Christoffersen, 2013).

More than a dozen parameterizations and frameworks for frontal ablation processes have been published over the last decade, yet a universal numerical representation for frontal ablation at marine-terminating glaciers continues to elude scientific consensus (Benn et al., 2017; James et al., 2014). Existing parameterizations include calving models (also known as *calving laws*), submarine melting models, and coupled calving/melting models that all predict either the terminus position or ablation rate. Most parameterizations for frontal ablation represent terminus dynamics only in terms of iceberg calving (Bassis & Walker, 2012; Benn, Hulton, & Mottram, 2007; Levermann et al., 2012; Mercenier et al., 2018; Van der Veen, 1996), due in part to historic underappreciation of submarine melting rates (Truffer & Motyka, 2016). More recently, attempts have been made to incorporate both iceberg calving and submarine melting into numerical glacier models (Morlighem et al., 2016; Todd et al., 2018) and even to formulate frontal ablation exclusively as a function of submarine undercutting (Luckman et al., 2015).

Most current models of the Greenland Ice Sheet employ simple iceberg calving models as boundary criteria to represent advance and retreat of the ice sheet marine terminus or changes in frontal ablation rate (Benn et al., 2017; Goelzer et al., 2018). These simple calving models rely upon inputs of glacier and fjord properties that are commonly measured via satellite platforms and available in ice sheet-scale models, including ice thickness, fjord water depth, ice stresses, and ice velocity. Calving models vary in complexity from fixed glacier extents or simple flotation thresholds to more sophisticated representations of both calving and submarine melting processes (Goelzer et al., 2018). The lack of consensus among ice sheet models on which terminus boundary criteria should be employed in part reflects the fact that the relative accuracies of existing calving models are largely unknown. Given the observed importance of frontal ablation processes, their inaccurate representation is likely to result in inaccurate perturbations to the force balance that governs ice flow and may at least partially explain the large disparity in present and future predictions of dynamic discharge among ice sheet models (Aschwanden et al., 2019; Fürst et al., 2015; Peano et al., 2017). Conversely, accurate simulation of terminus dynamics improves modeled mass flux estimates, as recently demonstrated by Haubner et al. (2018). When input with historical terminus positions, Haubner et al. (2018) found that the Ice Sheet System Model (ISSM) was able to reproduce past mass flux at Upernavik Isstrom in Greenland with high fidelity to observations. It is therefore expected that the incorporation of improved iceberg calving models in prognostic ice sheet models will increase confidence in projections of dynamic discharge from Greenland.

To date, calving model generation has outpaced comprehensive validation of existing models, and thus, relative model accuracies and uncertainties remain largely unquantified at the ice sheet scale (Benn et al., 2017). Each published calving model has successfully reproduced elements of terminus dynamics at either observed or idealized glacier settings, but methods of validation vary considerably among models. Some calving models have been validated against observational data sets consisting of measurements from several tidewater glaciers (e.g., Bassis & Walker, 2012; Levermann et al., 2012; Mercenier et al., 2018; Van der Veen, 1996; Vieli et al., 2001) while others have been evaluated via implementation in 1-D, 2-D, or 3-D ice flow models (Morlighem et al., 2016; Nick et al., 2010; Otero et al., 2010; Todd et al., 2018). Process-driven ice flow models test calving models via comparison of modeled and observed terminus changes (e.g., Cook et al., 2012); however, parameterizations of basal traction, ice rheology, and surface mass balance employed by models are often poorly constrained such that observations can potentially be reproduced using a combination of inaccurate parameterizations (e.g., Benn, Hulton & Mottram, 2007; Enderlin et al., 2013). Recent evaluations of the same calving parameterization in two different ice flow models (Choi et al., 2018; Todd et al., 2018) resulted in dissimilar assessments of calving performance. This suggests that components of ice flow models other than the terminus boundary condition impact the performance of the calving model under investigation. The use of observational data to evaluate calving models outside of a broader modeling framework avoids the confounding and unquantified influence of other model components by allowing the calving parameterization to be tested in isolation. Importantly, none of the existing calving models have been validated at the ice sheet scale using consistent data sets and methods. Thus, there is a need for a



thorough, observational evaluation of calving models to resolve the relative accuracies and uncertainties of these criteria at the ice sheet scale.

Previous calving model validations also feature differing methods of model calibration, which complicate the interpretation of model performance at the ice sheet scale. Each published calving model contains at least one free parameter that must be calibrated to observations. Most previous validation studies have calibrated calving model parameters to a set of time-varying observations from individual glaciers (Choi et al., 2018; Levermann et al., 2012; Morlighem et al., 2016, 2019; Van der Veen, 2002) or to observations from an array of glaciers, with one observation per glacier (Bassis & Walker, 2012; Mercenier et al., 2018). While calving models calibrated over small spatial or temporal scales often produce accurate terminus change simulations at individual glaciers (e.g., Morlighem et al., 2016), such evaluations do not inform the performance of these models at the ice sheet scale. Due to the high individuality and inconsistency of Greenland glacier response to climate (e.g., Bartholomaus et al., 2016), employing individually tuned parameter values for centennial-scale ice sheet projections is undesirable (Slater et al., 2019). Instead, ice sheet models seek simple boundary criteria with fixed parameters that are thoroughly calibrated across spatial and temporal scales, since such models likely offer more reliable prediction of terminus changes across the entirety of the Greenland Ice Sheet and over projected timescales. To date, there is no study that has performed a full calibration and evaluation of existing calving models across comprehensive spatial and temporal scales to inform the utility of these models for ice sheet projections.

The recent proliferation of ice velocity observations (e.g., Howat, 2017), ice elevation measurements (e.g., Porter et al., 2018), and improved bed elevation estimates (Morlighem et al., 2017) across Greenland enables such a calibration and evaluation of calving models. In this paper, we use observed and modeled data from the Greenland Ice Sheet to evaluate and intercompare six calving models. These six calving models, each with at least one loosely constrained free parameter, are either currently employed or are candidates for use in ice sheet models of the Greenland Ice Sheet (Goelzer et al., 2018). Using observations from a diverse and representative sample of 50 outlet glaciers encircling the ice sheet, we empirically calibrate the free parameter in each calving model by identifying the parameter value that minimizes the misfit between predicted and observed frontal ablation behaviors. We perform three calibration tests. First, we calibrate the six calving models to individual glaciers to examine spatial variability in optimal model parameters and to reveal the range of calibrated parameters needed for each model to reproduce observed terminus conditions across 50 sample glaciers. Second, we test the performance of the six calving models at the ice sheet scale by applying a single model configuration calibrated to the 50 sample glaciers as a whole. This test provides measures of ice sheet-wide misfit between modeled and observed frontal ablation quantities and allows us to quantify the relative accuracy and uncertainty associated with each calving model at the ice sheet scale. Lastly, we investigate the temporal stability of model calibration using temporally dense observations from four of the 50 outlet glaciers. Based on these three analyses, we make recommendations regarding the use of calving models as boundary criteria for Greenland Ice Sheet model simulations and identify patterns of calving model shortcomings that are targets for improvement.

2. Data and Methods

2.1. Iceberg Calving Models

We select six iceberg calving models from the literature to evaluate against observations from Greenland outlet glaciers. The models, variable names, and the ranges of tested parameters are presented in the Supporting Information, Tables S1 and S2. Three of the calving models are *calving position models*, in that they predict the most extended stable terminus position, and the other three models are *calving rate models*, in that they predict the rate of iceberg calving. The calving position models include two buoyancy-based terminus criteria—height above flotation (HAF; Van der Veen, 1996) and fraction above flotation (FAF; Vieli et al., 2001) —and a crevasse depth criterion (CD; Benn, Hulton, & Mottram, 2007; Nick et al., 2010). In both the HAF and FAF models, the buoyancy of the glacier exerts a first-order control on the location of the calving front. The glacier terminus is permitted to thin until it reaches a critical height above buoyancy, whereupon ice is predicted to calve off. In the HAF model, the critical ice thickness is the sum of the buoyancy height and a fixed height, h_c , such that the ice thickness *H* must satisfy the inequality,



$$H \ge H_b + h_c,\tag{1}$$

where $H_b = \frac{\rho_{sw}}{\rho_i}D$, taking *D* to be the fjord depth (i.e., thickness of the submerged portion of the glacier) and ρ_{sw} and ρ_i are densities of sea water and ice, respectively. The FAF model parameterizes the heightabove-buoyancy threshold as a fraction, *f*, of the water depth to account for observed differences in ice cliff height between thick, fast-flowing and thin, slow-flowing marine-terminating glaciers (Vieli et al., 2001):

$$H \ge H_b(1+f). \tag{2}$$

Since there is no theoretical derivation for the values of h_c and f, we treat them as free parameters that must be calibrated to observations. Although neither the HAF nor FAF model permit floating ice, the preponderance of grounded glaciers in Greenland and the continued use of floation criteria in ice sheet models motivate the inclusion of these calving models in this study.

The CD calving model assumes that the first-order control on terminus position is longitudinal stretching near the glacier terminus (Benn, Hulton, & Mottram, 2007). In response to extensional stresses, surface crevasses are assumed to penetrate to a depth where the longitudinal strain rate exactly balances the creep closure rate from ice overburden pressure (Nye, 1955, 1957),

$$d_s = \frac{\sigma}{\rho_i g},\tag{3}$$

where d_s is the depth of the crevasse, σ is the tensile stress responsible for crevasse opening, and *g* is gravitational acceleration. Consistent with Nick et al. (2010), we represent σ as the horizontal resistive stress, R_{xx} , which is defined as the full stress minus the lithostatic stress (Van der Veen, 2013, p. 56). The value of R_{xx} is calculated from observed 2-D strain rates using the constitutive equation (Glen, 1955) and Van der Veen (2013), such that

$$R_{\rm xx} = B\dot{\varepsilon_e}^n \frac{1-n}{n} (2\dot{\varepsilon_1} + \dot{\varepsilon_2}),\tag{4}$$

where *B* and *n* are the ice stiffness parameter and stress exponent, $\dot{\varepsilon}_e$ is the effective strain rate defined as $\dot{\varepsilon}_e^2 = \frac{1}{2}(\dot{\varepsilon}_1^2 + \dot{\varepsilon}_2^2)$, and $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ are the two strain rate eigenvalues. The presence of water in surface crevasses deepens crevasse penetration depths through added downward pressure exerted by the water, $P_w = \rho_w g d_w$ (Benn, Hulton, & Mottram, 2007). The predicted depth of surface crevasses therefore becomes

$$d_s = \frac{R_{xx}}{\rho_i g} + \frac{\rho_w}{\rho_i} d_w.$$
 (5)

In addition to surface crevasses, basal crevasses may form when a glacier is at or near flotation and longitudinal stretching rates are large. Again, from Nick et al. (2010), the propagation height of basal crevasses, d_b , is estimated from R_{xx} as

$$d_b = \frac{\rho_i}{\rho_{sw} - \rho_i} \left(\frac{R_{xx}}{\rho_i g} - H_{ab} \right),\tag{6}$$

where $H_{ab} = H - H_b$. Calving consequently occurs in the CD model where surface crevasses intersect the waterline, or where basal and surface crevasses intersect. The modeled glacier terminus is defined as the most seaward location where the following conditionals are both met:

$$h > d_s, \quad H > (d_s + d_b), \tag{7}$$

in which h = H - D.We treat the depth of the water in crevasses, d_w , as a free parameter, allowing the CD model to be calibrated empirically.



The three rate models include the eigencalving model (EC; Levermann et al., 2012), a von Mises criterion (VM) proposed by Morlighem et al. (2016), and a calving relation based upon the surface stress maximum (SM) introduced by Mercenier et al. (2018). The EC model predicts a calving rate proportional to the two strain rate eigenvalues near the glacier terminus,

$$u_c = K_{ec} \cdot \max(\dot{\varepsilon}_1, 0) \cdot \max(\dot{\varepsilon}_2, 0), \tag{8}$$

in which the eigenvalues are averaged over a stress coupling length near the terminus. The stress coupling length is calculated as 4.5 times the ice thickness, following empirical observations by Enderlin et al. (2016). Equation 8 implies that if either eigenvalue is compressional, calving is suppressed. The EC model relies upon the empirical proportionality constant, K_{ec} , for calibration to observed calving rates (Levermann et al., 2012).

The VM criterion assumes that iceberg calving is governed by the tensile stress regime at the glacier terminus. Predicted calving rate depends on the ice velocity, v, at the glacier terminus and is modified by the ratio of terminus tensile stress to a tensile stress threshold, σ_{max} ,

$$u_c = v \frac{\sigma_{vm}}{\sigma_{max}},\tag{9}$$

where the state of tensile stress at the terminus is given by the VM stress,

$$\sigma_{vm} = \sqrt{3} \cdot B \cdot \varepsilon_{ie}^{\frac{1}{n}}$$
(10)

In equation 10, $\dot{\varepsilon}_{te}$ is the effective tensile strain rate, $\dot{\varepsilon}_{te}^2 = \frac{1}{2} [\max(\dot{\varepsilon}_1, 0) + \max(\dot{\varepsilon}_2, 0)]^2$, averaged over the glacier stress coupling length nearest the terminus. When $\sigma_{vm} < \sigma_{max}$, the glacier will advance, and conversely, when $\sigma_{vm} > \sigma_{max}$, the glacier will retreat (Morlighem et al., 2016). At present, σ_{max} is considered to be a material property, so it is treated as a free parameter in our testing of the VM model. Note that the simple submarine melt parameterization that is coupled to the VM calving model in Morlighem et al. (2016) is left out of this study so that we may intercompare the performance of the VM calving criterion directly to other calving rate models.

Calving in the SM model is primarily a function of the extensional stresses at the glacier termini resulting from the hydrostatic ice cliff imbalance (Mercenier et al., 2018). It is assumed that a large crevasse forms near the glacier terminus where the principle stress reaches its maximum value (assuming tension is positive):

$$\sigma_1 = \rho_i g H (0.4 - 0.45(\omega - 0.065)^2), \tag{11}$$

where $\omega = \frac{D}{H}$. The location (*x_m*) of the principle stress at the glacier surface is defined relative to the terminus as a unitless fraction of the ice thickness and is approximated in terms of the fjord water depth and ice thickness,

$$x_m = 0.67(1 - \omega^{2.8}). \tag{12}$$

When full failure of the ice occurs at the location of x_m , it is assumed that the damaged ice is rapidly removed and x_m becomes the new location of the glacier terminus. Mercenier et al. (2018) propose that ice damage at x_m may be accomplished through a variety of processes, including basal crevassing, hydro-fracturing via surface water, or rapid elastic crevasse propagation. The time needed for these processes to achieve ice failure is given according to the isotropic damage relation (Pralong, 2006),

$$T_f = \frac{(1-D_0)^{r+k+1} - (1-D_c)^{r+k+1}}{(r+k+1)B_d(\sigma_1 - \sigma_{th})^r},$$
(13)

in which r, k, and B_d are damage constants, D_0 and D_c are initial and critical damage values, and σ_{th} is the



threshold stress that must be exceeded for damage initiation. Calving rate is thus expressed as a ratio of distance over time:

$$u_c = \frac{x_m H}{T_f} = \widetilde{B} \left(1 - \omega^{2.8} \right) \cdot \left(\sigma_1 - \sigma_{th} \right)^r H.$$
(14)

In equation 14, \widetilde{B} encompasses a constant term related to ice damage,

$$\widetilde{B} = \frac{0.67(r+k+1)B}{(1-D_0)^{r+k+1} - (1-D_c)^{r+k+1}}.$$
(15)

A more complete explanation of variables is given in Mercenier et al. (2018). The SM model contains three empirical parameters: \tilde{B} , r, and σ_{th} and is reported to not be very sensitive to the exact choice of parameter values within a reasonable range (Mercenier et al., 2018). Therefore, we set r = 0.5, $\tilde{B} = 65$ MPa^{-r}a⁻¹, and select σ_{th} as our free parameter for model calibration since it has the simplest physical translation.

We endeavor here to evaluate many of the most well-known and broadly applied calving models. However, our goal to test models against observations precludes the evaluation of several existing calving models for which we lack necessary environmental data of sufficient resolution or that which yield unrealistic results, notably the undercutting model by Luckman et al. (2015) and the yield strength criteria proposed by Bassis and Walker (2012). Additionally, while models of submarine melting now exist (Rignot, Xu, et al., 2016; Slater et al., 2017; Xu et al., 2013), sufficient uncertainty in both the physical structure of these models and their uncertainties persist, such that we cannot be confident of their utility at ice sheet scale (Slater et al., 2018; Wagner et al., 2019). The six models tested in this study were designed only to represent mechanical calving processes and do not include explicit representation of submarine melting. However, since the six models are empirically calibrated to observations of total frontal ablation, submarine melting is implicitly accounted for in the calibrated model configurations. While it would be preferable to test the three calving rate models against direct measurements of calving rates alone and not frontal ablation rates, ice sheet-wide partitioned calving rate estimates (as distinct from submarine melting) are presently unavailable for use in such a validation. Instead, we rely upon implicit representation of submarine melting to model frontal ablation using a bulk, inclusive calving model, as is commonly done in many ice sheet models (Goelzer et al., 2018).

2.2. Observational Data

The six calving models under investigation collectively rely on four fundamental data sets for model inputs and evaluation: bed elevation, ice surface elevation, ice surface velocity, and observed terminus position. Using data from approximately one quarter of all marine-terminating outlet glaciers in Greenland, we test calving models on their ability to account for variability in glacier terminus change across a range of spatial and temporal scales.

Ice thickness data are derived from differencing ice elevation measurements obtained from NASA's Operation IceBridge Airborne Topographic Mapper (ATM) LiDAR and WorldView satellite stereo imagery-derived digital elevation models (DEMs) with bed elevation estimates from Greenland BedMachine v3 (Morlighem et al., 2017). Operation IceBridge ATM data have horizontal resolution of better than 1 m and vertical accuracy of 0.07 cm (Martin et al., 2012). High-resolution (2 m) DEMs constructed from DigitalGlobe's WorldView-1 and WorldView-2 satellite images as part of the ArcticDEM project (Porter et al., 2018; http://data.pgc.umn.edu/elev/dem/setsm/ArcticDEM/geocell/v3.0/2m/) capture the lower ~5–10 km of outlet glaciers with approximately 3 m vertical uncertainty (Enderlin et al., 2014; Noh & Howat, 2015). BedMachine v3 combines radar-constrained mass conservation modeling with bathymetric measurements around the ice sheet periphery to create a seamless, 150-m resolution map of Greenland bed elevations (Morlighem et al., 2017; https://nsidc.org/data/IDBMG4). Errors in bed elevation at our sample glaciers vary from less than 50 m at well-sampled glaciers to over 150 m.

Ice surface velocities are extracted from speckle tracking of satellite imagery from both optical (Howat et al., 2017) and radar (Joughin et al., 2010) sensors from NASA Making Earth System Data Records for Use in Research Environments (MEaSUREs) products (NSIDC; http://nsidc.org/data/



nside-0481; http://nside.org/data/nside-0646). These 100-m resolution velocity maps capture the fast-flowing trunks and termini of Greenland outlet glaciers with systematic errors estimated at 3% of the velocity for radar-derived velocities (Joughin et al., 2010). Ice velocity maps corresponding to each ice elevation profile or WorldView DEM are generally from within 30 days of the ice elevation observation, with 90% from within 40 days. For ATM elevation measurements from late April and May, the corresponding ice velocities are all from May or June due to the absence of velocity maps during early spring. For the velocity measurements corresponding to WorldView DEMs, 68% straddle the DEM date, while 10% precede the DEM date and 22% succeed the DEM date. We quantify potential biases arising from temporal mismatch between ice elevation and ice velocity measurements by adjusting our thickness estimates using a glacier thinning rate of $\frac{\partial H}{\partial t} = -0.1 \text{ m d}^{-1}$ during summer months (May through September) and $\frac{\partial H}{\partial t} = 0.05 \text{ m d}^{-1}$ during nonsummer months according to observations from Helheim Glacier and Kangerdlussuaq Glacier (Kehrl et al., 2017) and performing the same calving model calibrations and performances when input with time-corrected ice thicknesses; therefore, we only report results using original ice thickness and velocity measurements.

We convert surface strain rates derived from ice flow speeds to ice stresses using n = 3 and the temperature-dependent rate parameter, B = 324 kPa⁻¹ yr^{-1/3}, that corresponds to an ice temperature of -5° C (Cuffey & Paterson, 2010). Though the depth-averaged ice temperature of most glaciers in Greenland is potentially colder than -5° C, borehole observations suggest that the ice in surface and basal regions where crevasses are expected to form near glacier termini may generally be in the range of -2 to -10° C (Iken et al., 1993; Lüthi et al., 2002).

We calculate observed frontal ablation rates as the difference between glacier length change over time and ice velocity,

$$u_f = u_i - \frac{\Delta l}{\Delta t},\tag{16}$$

where $\frac{\Delta l}{\Delta t}$ is the distance between two traced terminus margins from satellite images divided by the time between satellite images and u_i is the ice velocity. We define u_f as a positive quantity oriented up-glacier for historical reasons, opposite the convention for positive u_i and $\frac{\Delta l}{\Delta t}$. The two satellite images for terminus traces are coincident with or fall within the dates of the two satellite images employed in the generation of the corresponding ice velocity map. Terminus margins are digitized by hand from Landsat and Sentinel satellite images using the Google Earth Engine digitization tool developed by Lea (2018). The change in time between terminus traces is typically 11–35 days, depending on the corresponding velocity observation period and the availability of suitable satellite imagery. Similar to Luckman et al. (2015), we calculate length change rate and ice velocity along a series of 20 parallel profiles oriented with ice flow and spaced 100 m apart that span the center 2,000 m of the terminus width. By averaging frontal ablation rates across the middle 2,000 m of the glacier, we reduce the effect of large individual calving events that only impact part of the terminus and instead obtain a frontal ablation rate that represents the behavior of the glacier where it is thickest and fastest. Errors arising from digitization of satellite images are estimated to be on the order of 30 m (Carr et al., 2017), which, combined with reported ice velocity errors, results in frontal ablation rate uncertainties of <1.5 m d⁻¹.

Observed terminus positions used to evaluate calving position models are identified as the calving cliff crest, as opposed to the intersection of the calving cliff and the waterline. We frequently observe in the ATM surface elevation measurements from April/May that sample glaciers terminate in a several-hundred-meter ramp of ice blocks sloping down from the cliff top to the waterline. Since we do not expect this ice ramp to contribute to the glacier stress balance in the same manner as intact glacier ice, it is not included in our evaluation of calving position models.

Input data for the six calving models are interpolated onto 1-D center profiles that follow ATM flight paths up the middle of glaciers, thus capturing the behavior of the glacier where it is generally thickest and fastest.





Figure 1. Sample glacier locations in Greenland shown in (a). Black circles represent glaciers with one observation available that contribute to spatially distributed data set. Red triangles are labeled with glacier abbreviations and denote glaciers that have an additional 15 observations that contribute to the temporally distributed data set with observation dates shown in (b).

Measures of calving model accuracy are obtained through comparison between modeled calving and observed frontal ablation quantities (positions or rates) along the 1-D profiles. Our 1-D validation approach is consistent with methods employed in studies quantifying terminus change at outlet glaciers in Greenland (e.g., Bevan et al., 2012) as well as recent calving model validation studies (Choi et al., 2018; Morlighem et al., 2019). Observations of dynamic changes at marine-terminating glaciers show that on interannual timescales, terminus retreat and stabilization frequently initiate from the center (Catania et al., 2018; McNabb & Hock, 2014), which suggests that capturing glacier behavior along the centerline is critical for accurate representation of dynamic changes. Furthermore, terminus change studies reveal that the position of the terminus rarely varies by >1 km across its entire width (Catania et al., 2018; Murray et al., 2015), a difference that is currently below the highest resolution Greenland Ice Sheet models participating in the Ice Sheet Model Inter-comparison Project 6 (ISMIP6; Goelzer et al., 2018). For these reasons, we expect that our 1-D profile methods sufficiently represent dynamic terminus processes for our purposes.

Operation IceBridge ATM ice elevation data cover nearly 100 glaciers, but the requirements of near-coincident ice elevation and ice velocity data for glaciers with reasonably accurate bed elevation estimates significantly reduce the number of outlet glaciers in Greenland with sufficient data for our analysis. Of the roughly 200 outlet glaciers in Greenland, we identify 50 glaciers that have a complete set of suitable observations against which to evaluate calving models (Figure 1a). We perform a more detailed intraannual analysis on four well-studied glaciers with WorldView DEMs created as part of the Arctic DEM project (Porter et al., 2018) to assess the effects of temporal sampling on our analysis.

We evaluate each calving model against terminus change across spatial and temporal scales using a set of spatially distributed observations and a set of temporally distributed observations. The spatially distributed observation set consists of a single set of measurements from all 50 sample glaciers. These glaciers span a range of glacier characteristics and environmental settings, exemplified by varying ice thicknesses (Howat et al., 2014), ice velocities (Joughin et al., 2010), calving styles (Fried et al., 2018; Veitch & Nettles, 2012),





Figure 2. Diagram of example misfit measurement at Rink Glacier for calving position models shown with example criteria of HAF model and critical excess ice height, $h_c = 50$ m. Red shading depicts unstable glacier ice that is assumed to calve off according to the HAF criteria. Modeled and observed terminus positions are interpolated onto center profile line (shown in yellow) such that misfit distance is represented by 1-D offset between points. Background is Landsat 8 image from 03 May 2016.

air temperatures (which affect both ice temperature and melt rates; Cowton et al., 2018), ocean water temperatures (Straneo et al., 2012), and recent position changes (Murray et al., 2015), such that our 50 sample glaciers broadly represent the marine boundary of the Greenland Ice Sheet as a whole. Although glaciers from all sectors of the ice sheet are included in our 50-glacier sample, most of our sample glaciers are concentrated in the NW and SE sectors of the ice sheet, which are sectors that combine to account for approximately 80% of the total ice sheet frontal ablation flux (Enderlin et al., 2014). Observations at sample glaciers consist of bed elevation and coincident ice elevation and ice velocity observations that are from May/June and span the years 2009 to 2017.

The temporally distributed observation set consists of 15 observations each from Jakobshavn Isbræ, Kangerdlussuaq Glacier, Illullip Sermia, and Daugaard-Jensen Glacier for a total of 60 observations. Collectively, these observations span the winter, summer, and shoulder seasons during the years 2010 to 2017 as shown in Figure 1b. Frontal ablation rates at outlet glaciers vary from virtually zero in winter to the loss of several kilometers of ice in a single day during early summer (Amundson et al., 2010; Robel, 2017). It is therefore expected that the performance of a calving model calibrated during May/June may share little similarity with the performance of the same model during other seasons. The four glaciers selected exhibit some of the largest seasonal variability in observed frontal ablation rates and terminus position in Greenland (Bevan et al., 2012; Cassotto et al., 2015; Kehrl et al., 2017; Moon et al., 2015; Schild & Hamilton, 2013) and thus provide a challenging test for the calving models.

2.3. Calving Model Calibration and Evaluation

We seek to identify the best value of each model's free parameter by testing models with parameters drawn from a wide range of values that are inclusive of those recommended by the literature. We also assess model performance at broad spatial and temporal scales. We first draw on each observation of the 50 individual sample glaciers in the spatially distributed data set to empirically calibrate each calving model. We refer to the parameter that yields the most accurate terminus position or calving rate for each observation as the *observation-optimized* parameter. For calving rate models, model accuracy is measured according to the *misfit rate*, defined as the difference between modeled calving rates and observed frontal ablation rates. The accuracy of calving position models is determined by the 1-D along-profile distance between modeled and observed terminus positions, referred to as the *misfit distance* (Figure 2).



Calving models are then calibrated to the entirety of the Greenland Ice Sheet by identifying *spatially optimized* parameter values that minimize the total misfit between model predictions and observations at all 50 sample glaciers. This calibration process reflects the current need in ice sheet models for a single, calibrated, universal boundary criterion that may be applied to the entire Greenland Ice Sheet. Spatially optimized parameter values are defined as those that yield a median of individual glacier misfits nearest to 0 m for calving position models and 0 m d⁻¹ for calving rate models. Using spatially optimized parameter values, each calving model configuration underpredicts the calving position or calving rate at 25 glaciers and overpredicts the calving position or calving rate at the other 25 glaciers, hence offering the best compromise in model performance across all 50 sample glaciers.

Lastly, we investigate how calibrated parameter values vary through time at individual glaciers and, consequently, how representative the spatially optimized model configurations are of ice sheet terminus dynamics over time. For each calving model, we identify observation-optimized parameter values for the four sample glaciers in the temporally distributed observation set. We then determine *temporally optimized* parameter values for each calving model and for each glacier in the temporally distributed observation set. Temporally optimized parameter values are defined as the values that yield a median misfit nearest to 0 m for calving position models and 0 m d⁻¹ for calving rate models across all 15 observations at a given sample glacier.

We assess the performance of spatially optimized and temporally optimized calving models in terms of model bias, model uncertainty, and model sensitivity. We define model bias as the median modeled misfit (m_{50}) , and the spatially optimized and temporally optimized models are expected to have zero bias as a result of our optimization process. Model uncertainty measures the spread of all glacier misfits around the median misfit value. We assess model uncertainty for calving rate models via the 25th and 75th percentiles (m_{25} and m_{75}) of rate misfit such that model bias and uncertainty are given in the form $\delta_{model} = m_{50} \pm m_{75} - m_{50} + m_{75} - m_{50} + m_{50} = 0 \text{ m d}^{-1}$, δ_{model} reduces to $\delta_{model} = \pm m_{75} + m_{50} + m_{50}$ models, we can determine an optimized parameter that results in a median misfit of 0 m, but our analysis is unable to quantify misfits associated with predicted termini down-fjord of the input observation domain. Although misfit rates can be positive or negative, only positive misfit distances (indicative of model-predicted retreat) are possible when testing calving position models against observed glacier geometries because there are no thickness and speed observations down-fjord of termini. Based on extrapolated, near-linear sensitivities of position models to parameter change, we therefore make the assumption that overadvanced misfit distances are comparable in magnitude with the overretreated misfits such that $\delta_{model} = m_{50} \pm (m_{75} - m_{50})$ m d⁻¹. More complete quantification of overadvanced misfits requires realistic numerical extrapolation of glacier extents down-fiord—a process that involves precise knowledge of the local stress balance across 50 outlet glaciers and is thereby beyond the scope of this study. However, the prevalence of shallow sills followed by deeper water depths down-fjord of current glacier fronts likely constrains the potential for large dynamic advances at many outlet glaciers (Rignot, Fenty, et al., 2016) and thus provides a physical basis for our assumption regarding overadvanced misfit distances.

Finally, we calculate model sensitivity, a measure of the change in model bias resulting from a small change in free parameter value. In the likely occurrence that our analysis does not represent a perfect characterization of spatially and temporally optimized model parameters, we seek a model that will be relatively insensitive to small perturbations in parameter value. We define this model sensitivity as the change in model bias resulting from a small parameter change, taken to be 25% of the interquartile range of all observation-optimized parameter values included in our analysis (110 total values). In combination, model sensitivity and model uncertainty provide thorough and robust metrics by which to intercompare the performance of the six tested calving models.

We design our analysis to minimize the effect of observational uncertainty on our results. Our selection of 50 sample outlet glaciers contains only those glaciers with high-quality input data sets. Nonetheless, potential errors associated with bed elevation estimates, observed frontal ablation rates, and temporal mismatch between surface elevation and ice velocity measurements add uncertainty to our results. At individual glaciers, if we treat uncertainties as systematic biases rather than random errors, the observation-optimized parameter values can deviate by more than a factor of 2 in the most extreme cases. However, when we



consider the parameters optimized to the aggregate observation data sets (i.e., the spatially or temporally optimized parameters), the inclusion of observational uncertainties changes optimal parameter values for the six calving models by ~30% and associated model uncertainties by <10%. Additionally, given that our large, highly diverse set of outlet glaciers have observation-optimized parameter values that vary by a factor of 10 or more, we assume that the propagation of input data errors has a trivial impact on the results of our glacier ensemble and the pattern of our study's main findings.

3. Results

3.1. Spatial Variability in Observation-Optimized Calibrations

We first determine observation-optimized parameters for each of the 50 sample glaciers by identifying the free parameter value that optimizes each fit between modeled calving and observed frontal ablation behaviors. We find that, through varying calving model parameter values, we are able to accurately reproduce terminus conditions (i.e., with zero misfit) at almost all glaciers with five out of six calving models tested here (Figures 3 and 4). We furthermore find that calving model accuracy at individual glaciers is highly sensitive to parameter value.

3.1.1. Calving Position Models

The HAF, FAF, and CD models all reproduce most sample glacier terminus positions with high accuracy when using a model parameter tuned to each individual glacier. We find that 9 of the 50 sample glaciers feature short (<2 km), ungrounded, terminal extensions in the ATM ice elevation measurements from April/May, while 2 sample glaciers terminate in floating ice tongues (i.e., >2 km ungrounded ice). The other 39 glaciers are grounded between 1 and 80 m above flotation or with a water depth fraction of 0.01 and 0.4 (Figures 3a and 3b). Perfect simulation of terminus position by the HAF and FAF models is therefore achieved at 39 of the 50 sample glaciers using observationally optimized parameter values, though we note that for seven of the 11 ungrounded termini, the modeled termini positions are less than 1 km from the observed positions. The HAF and FAF models perform particularly poorly when applied to glaciers terminating in floating ice tongues such as at Kangerdlussuaq Glacier and Petermann Glacier, which feature floating ice tongues of approximately 5 and 60 km in length, respectively. Observationally optimized h_c (critical height above buoyancy) and f (fraction above flotation) parameter values at glaciers in northern Greenland are generally smaller than values for glaciers in southern Greenland, a pattern that is broadly consistent with the occurrence of ungrounded ice at northern outlet glaciers.

The CD model exactly reproduces terminus position at 48 of 50 sample glaciers, irrespective of flotation state. Water depth (d_w) values required to reproduce observed terminus positions range from 0 up to 100 m across the 50 sample glaciers (Figure 3c). In the case of Kangerdlussuaq Glacier and Petermann Glacier, the CD model can reproduce observed terminus positions if it is assumed that surface crevasses are filled with 26 m of water at Kangerdlussuaq Glacier and 43 m at Petermann Glacier. No obvious spatial pattern exists in the distribution of observationally optimized d_w values at the 50 sample glaciers, though the largest d_w values are associated with glaciers located in southern Greenland (Figure 3c).

3.1.2. Calving Rate Models

As with the calving position models, we compare modeled calving rates with observed, ~ 6-week average frontal ablation rates from May/June at all sample glaciers to determine how well calving rate models can simulate frontal ablation rates in Greenland. Observed frontal ablation rates vary from 0 to 69 m d⁻¹, suggesting that these observations are from a time when frontal ablation at glacier fronts is highly variable across the ice sheet. While frontal ablation is low at some glaciers, such as Petermann Glacier, vigorous calving has commenced at other glaciers, as at Zachariae Isstrom. Accounting for the observed variability in frontal ablation rates across Greenland challenges the three calving rate models investigated in this study, while the use of ~6-week average frontal ablation rates ensures that individual large calving events do not unduly influence the measured frontal ablation rates.

We find that the VM model can accurately predict frontal ablation rates from all 50 sample glaciers. However, at eight sample glaciers, accurate simulation of frontal ablation rates by the VM model is only achieved using extremely high (>5 MPa) values for σ_{max} . The VM model struggles in cases where observed ice velocities are on the order of tens of m d⁻¹, but observed frontal ablation rates are near 0 m d⁻¹, since





Figure 3. Spatial variability in observation-optimized parameter values for six calving models at glaciers around Greenland in (a–f). Parameter h_c is critical height above bouyancy, *f* is water depth fraction, d_w is crevasse water depth, σ_{max} is ice tensile strength threshold, σ_{th} is stress threshold for ice damage initiation, and K_{ec} is proportionality constant. Eigencalving model in (f) is only valid at nine glaciers around Greenland due to the model requirement that both strain rate eigenvalues must be positive to allow calving.

such a condition requires that σ_{max} greatly exceeds the tensile VM stress at the terminus, σ_{vm} , in order that the VM modeled calving rate approach zero. The SM model accurately reproduces frontal ablation rates at 38 of 50 sample glaciers, but overestimates frontal ablation rates at five glaciers and underestimates the frontal ablation rates at the remaining seven glaciers. For these 12 glaciers individually, there is no value of the ice strength threshold parameter σ_{th} that yields zero misfit.





Figure 4. Calving position models in (a–c) and calving rate models in (d–f) showing calibration of free parameters to 50 sample glaciers. Red line tracks the median misfit, black lines bracket model uncertainty, defined as interquartile range in (d–f) and 75th percentile in (a–c). Light gray lines depict calibration curves for each individual glacier. Ice sheet-wide optimal parameters for each model are identified as the parameters which yield a median misfit of 0.

The SM model is markedly less sensitive to the choice of free parameter value than the VM model, which may contribute to its inability to reproduce exact frontal ablation rates at 12 of 50 glaciers. Variation of model parameters other than that which we explored here (see section 2 and Mercenier et al., 2018) has the potential to further improve the model fit. Modeled calving rates in the SM model for individual glaciers exhibit a smaller range than for the VM model since the maximum modeled calving rate for a particular glacier with the SM model is a predetermined function of that glacier's H/D ratio and thus cannot be enhanced by any value of the stress threshold for ice damage initiation, σ_{th} . The spatial distributions of optimal σ_{max} and σ_{th} values at sample glaciers do not reveal any obvious spatial pattern, but suggests that most observationally optimized σ_{th} values lie within a narrower range than optimal σ_{max} values (Figures 3d and 3e).

The EC model exactly reproduces observed frontal ablation rates at nine of 50 sample glaciers using free parameter values that vary by three two orders of magnitude, between 2×10^4 m a to 1.3×10^6 m a (Figure 3f). Due to the requirement that there must be tensile stretching in both principle directions at the glacier terminus to induce calving, the EC model is invalid at 41 of 50 sample glaciers. This is not a surprising finding given that most Greenland outlet glaciers are confined to narrow fjords that suppress transverse ice flow. For the nine glaciers where the EC model is valid, the sensitivity of modeled calving rates to the choice of parameter K_{ec} enables the EC model to account for highly variable frontal ablation rates. However, the wide span of observationally optimized values for K_{ec} makes it unlikely that a single K_{ec} value could allow accurate simulation at all nine glaciers (Figure 3f). Given the limited applicability of the EC model to Greenland outlet glaciers, we do not include the model in subsequent results.



Table 1 Performance of Spatially Optimized Calving Models to 50 Sample Glaciers					
Model	HAF	FAF	CD	VM	SM
Spatially optimized	$h_c = 11 \text{ m} [7 \text{ to } 19]$	f = 0.04 [0.01 to 0.08]	$d_w = 24 \text{ m} [22 \text{ to } 24]$	$\sigma_{max} = 0.45 \text{ MPa} [0.36 \text{ to } 0.52]$	$\sigma_{th} = 0.33 \text{ MPa} [0.28 \text{ to } 0.46]$
Model uncertainty	± 0.9 km [0.9 to 1.2]	± 1.3 km [1.2 to 1.3]	± 0.3 km [0.3]	$\pm_{4.2}^{2.4} \text{ m d}^{-1} \left[\pm_{3.8}^{2.4} \text{ to } \pm_{5.2}^{3.8}\right]$	$\pm_{3.5}^{8.0} \text{ m d}^{-1} [\pm_{3.9}^{2.4} \text{ to } \pm_{6.6}^{8.0}]$

Note. Bracketed parameter values and model uncertainty show the ranges that exist when uncertainties in input ice thickness and observed frontal ablation rates are taken into account.

3.2. Spatially Optimized Calibrations

The empirical calibration of each model to the collective set of 50 sample glaciers reveals the single optimal parameter value within each model that minimizes the overall misfit at the ice sheet scale. With the exception of the EC model, we find that all calving models investigated in this study can be successfully calibrated to produce a median misfit of 0 using a fixed, spatially optimized parameter value for all 50 sample glaciers (Figure 4). The CD model exhibits the lowest model uncertainty among the tested position models while the VM model features the lowest model uncertainty among the calving rate models. Parameter values and associated model misfits are given in Table 1.

3.2.1. Calving Position Models

Using observations from our 50 sample glaciers, we find that the three calving position models all perform well at the ice sheet scale. The HAF model with $h_c = 11$ m estimates terminus position at 50% of the sample glaciers to within 0.9 km which is slightly better than the FAF model with f = 0.04 (1.3 km for 50% of sample glaciers). Expectedly, the HAF and FAF models exhibit identical deficiencies at glaciers featuring floating termini, such as Kangerdlussuaq Glacier and Petermann Glacier (Figures 5a and 5b). The HAF model notably outperforms the FAF model at several of Greenland's largest glaciers, including Jakobshavn Isbræ, Helheim Glacier, and Zachariae Isstrom. These glaciers are all relatively thick and close to buoyancy, such that the ice thickness fractions above buoyancy (f) are relatively small near the glacier termini while the heights above buoyancy (h_c) are comparably larger. The CD model with $d_w = 24$ m estimates the terminus position at 50% of the sample glaciers to within 0.3 km and 90% to within 1 km (Figure 5c). Only two glaciers, Issuussarsuit Sermia and Humboldt Gletcher, in the NW of Greenland exhibit modeled terminus positions more than 1 km away from the observed terminus positions when $d_w = 24$ m (1.3 and 1.2 km, respectively). We note from Figure 5c that the spatially optimized CD model has low model sensitivity to parameter variation, as indicated by the low slope of the median misfit (Figure 4c): with an approximate doubling of crevasse water depths to $d_w = 50$ m, the CD model still simulates terminus positions at 50% of sample glaciers to within 1 km. Expectedly, the CD model outperforms the HAF and FAF models at glaciers with floating termini, but the CD model also proves to be more accurate than the floation models at almost all grounded sample glaciers as well, with the exception of three glaciers in the Northwest of Greenland.

3.2.2. Calving Rate Models

The VM calving model with a tensile stress threshold $\sigma_{max} = 0.45$ MPa reproduces observed frontal ablation rates at 50% of sample glaciers to within an uncertainty of $\pm_{4.2}^{2.4}$ m d⁻¹ and at 90% of sample glaciers to within $\pm_{11.1}^{12.8}$ m d⁻¹ (Table 1). The ice sheet-wide accuracy of the VM model is similar for σ_{max} values between 0.4 and 0.5 MPa, though model bias and uncertainty both increase considerably outside of this range (Figure 4d). The SM model with a spatially optimized σ_{th} value of $\sigma_{th} = 0.33$ MPa reproduces 50% of observed frontal ablation rates to within uncertainty of $\pm_{3.5}^{8.0}$ m d⁻¹ and reproduces 90% of observed rates to within $\pm_{21.5}^{13.7}$ m d⁻¹. SM model bias is notably not very sensitive to increases in the value of σ_{th} , likely owing to fact that modeled calving rates become 0 m d⁻¹ when the principle ice stress σ_1 no longer exceeds the stress threshold for damage (σ_{th}). The spatially optimized VM and SM models underestimate frontal ablation rates at Zachariae Isstrom by more than 50 m d⁻¹ and at Ingia Isbræ by more than 20 m d⁻¹. We also observe that both models underestimate calving rates at glaciers in central west Greenland where observed frontal ablation rates are relatively high (15 to 30 m d⁻¹), but accurately estimate calving rates at glaciers in the North/Northwest of Greenland where observed frontal ablation rates are less than 10 m d⁻¹





Figure 5. Model misfits at individual glaciers using spatially optimized model configurations. Calving position models in (a–c) and calving rate models in (e) and (f). Panel (d) shows observed frontal ablation rates during May/June. Note that symbol size scales in (a–c) are equivalent. The exceptionally large (69 m d^{-1}) observed frontal ablation rate in northeast of Greenland belongs to Zachariae Isstrom glacier.

(Figures 5e and 5f). These model performance patterns are in contrast with the absence of spatial patterns exhibited by the spatially optimized calving position models (Figures 5a and 5c).

3.3. Temporal Variability in Observation-Optimized Calibrations

The positions of outlet glacier termini in Greenland typically fluctuate by <1 km per year (Bartholomaus et al., 2016; Catania et al., 2018; Moon et al., 2015) but interannually can change by up to several kilometers





Figure 6. Temporal variability in individual glacier optimal model parameters. Colored symbols represent observationally optimized parameter value calibrated against glacier observations that vary in season. Peak glacial runoff period from June through September is shaded gray. Filled black symbols depict the observationally optimized parameter values from May/June (section 3.1) and are shown for comparison.

(e.g., Cassotto et al., 2015). We observe that short-term (submonthly to monthly) frontal ablation rates typically vary from 0 to 50 m d⁻¹ over seasonal timescales for the four large glaciers included in this portion of the analysis, though we observe several short-term frontal ablation rates in excess of 100 m d⁻¹ at Jakobshavn Isbræ and Kangerdlussuaq Glacier. The four glaciers in our temporally distributed data set do not exhibit any sustained interannual dynamic changes, with the exception of a slight (<3 km) advance at Jakobshavn Isbræ over the period 2011 to 2017 (Khazendar et al., 2019). Similarly, we do not observe evidence of interannual variability in observed frontal ablation rates at the four glaciers in the temporally distributed data set, though we note that such variability is widespread in recent observations of Greenland (Moon et al., 2012; Howat & Eddy, 2012; Csatho et al., 2014).

Optimal model parameter values change to meet the seasonal and interannual changes in terminus position and frontal ablation rates in our observation data, as shown in Figure 6. Therefore, it is unclear whether the spatially optimized model parameters we identify using measurements during May/June are the most accurate parameter values to use during a different time of year or during a different year altogether. We find that temporal variability in observation-optimized model parameters at four individual glaciers is considerable and similar in magnitude to the observation-optimized parameter variability observed across the entire ice sheet.



3.3.1. Calving Position Models

Observationally optimized values for h_c and f exhibit similar variability over time for Jakobshavn Isbræ, Illullip Sermia, Daugaard-Jensen Glacier, and Kangerdlussuaq Glacier. Independent of season, observation-optimized h_c and f values vary by roughly 20 m and by fractions of 0.03 to 0.06, respectively, at all glaciers except Kangerdlussuaq Glacier, which features a floating ice tongue for the duration of the observation period and therefore has consistent optimized values of $h_c = f = 0$. Comparatively, the CD model exhibits greater variability in observational-optimized parameter values, with optimized d_w values fluctuating by more than 40 m at all four glaciers. Importantly, we find that optimized d_w values vary independently of season, a result that is in conflict with the physical meaning of d_w as the crevasse water depth, which is expected to vary seasonally as runoff is routed to surface crevasses. Observationally optimized parameters from the spatial validation data set (Figure 3) are shown in black symbols in Figure 6. For the three position models, these calibrated parameters are within the range exhibited by observationally optimized parameters from the temporal validation data set. This indicates that for position calving models, spatially optimized parameter values that are empirically calibrated to observations from May/June are likely to be broadly representative of calving conditions on annual timescales.

3.3.2. Calving Rate Models

Observation-optimized parameter values calibrated to temporally distributed observations for the VM model span the full range (0 to >5 MPa) of tested parameter values at three of the four sample glaciers in the temporal data set (Figure 6d). That is, the observation-optimized values of σ_{max} over 15 observation dates at a single glacier are equal in spread to that exhibited by the observation-optimized σ_{max} values across all 50 sample glaciers. Despite this temporal variability, there is a discernible seasonal influence on observationally optimized parameters at Jakobshavn Isbræ and Illullup Sermia. The smallest σ_{max} values at Jakobshavn Isbræ are found during estimated peak runoff months, which are coincident with peak observed frontal ablation rates. Similarly, the minimum optimal σ_{max} values for Illullip Sermia are all found during August, coincident with the largest observed frontal ablation rates. Observed frontal ablation rates at Kangerdlussuaq Glacier are almost all either exceptionally large ($u_f > 50 \text{ m d}^{-1}$) or negligibly small ($u_f < 2 \text{ m d}^{-1}$), suggesting the occurrence of kilometer-scale calving events punctuated by periods of quiescence. Correspondingly, observationally optimized σ_{max} values at Kangerdlussuaq Glacier exhibit a dichotomy between extremely large ($\sigma_{max} > 5 \text{ MPa}$) and small ($\sigma_{max} < 1 \text{ MPa}$) values, irrespective of observation month.

Observational-optimized σ_{th} values in the SM model span the full tested parameter range (0 to 0.5 MPa) for all four glaciers in the temporal validation data set (Figure 6e). As is the case with the σ_{max} parameter in the VM model, the temporal variability in observational-optimized σ_{th} values in the SM model at any of the four temporal set glaciers is equal to the spatial variability in observationally optimized σ_{th} values across all 50 glaciers. We likewise see evidence of a weak seasonal trend in optimal σ_{th} values at Jakobshavn Isbræ, where the largest calibrated σ_{th} values are found during minimal runoff months and the majority of low σ_{th} values are found during the peak runoff months. Likewise, low σ_{th} values for Illullip Sermia are all found during August. No obvious seasonal trend is visible in optimized parameters for either the VM or SM models at Daugaard-Jensen Glacier.

3.4. Performance of Spatially Optimized Parameters on Temporally Varying Data

Since we find that temporal variability in model parameters is approximately equal to or exceeds spatial variability in model parameters, it is not immediately clear how effective a spatially optimized calving model will be at simulating terminus dynamics over time at individual glaciers. To address this, we test calving models with spatially optimized model parameters against 15 temporally varying observations from four glaciers (Figure 7). We find that the HAF and FAF models exhibit model errors at individual glaciers that vary systematically with glacier retreat and advance, while the VM and SM models exhibit model errors that vary systematically with observed frontal ablation rate. Errors in the CD model are clustered near 0 km and appear uncorrelated with dynamic terminus changes.

The spatially optimized HAF model with $h_c = 11$ m performs well at Jakobshavn Isbræ and Illullip Sermia glaciers over time (Figure 7a). Although the terminus position for Jakobshavn Isbræ fluctuates by more than 5 km across the 15 temporally varying observations, model errors never exceed 1 km. However, at Kangerdlussuaq Glacier and Daugaard-Jensen Glacier, both of which primarily feature floating termini,





Figure 7. Performance of spatially optimized calving models using parameter from Table 1 against temporally varying observations from Daugaard-Jensen, Illullip Sermia, Jakobshavn Isbræ, and Kangerdlussuaq Glacier. Relative terminus positions in (a-c) are deviations from the median terminus position for each glacier across 15 observations. Misfits less than 0 km (down-fjord misfits) are depicted by "?" symbols since we cannot quantify negative misfit values for these relative terminus positions. There is no significance to the *y* axis position of plotted "?" symbols. Note difference in *y* axis scale between (a), (b), and (c).

model misfits are mostly in excess of 1 km and vary systematically with terminus position. Glacier retreat is associated with improved model misfits as an ungrounded terminus retreats toward the grounding line. Similarly, for the spatially optimized FAF model employing f = 0.04, model misfits are small for the grounded glaciers Jakobshavn Isbræ and Illullip Sermia and are uncorrelated with terminus position changes (Figure 7b). The FAF model exhibits large and consistent errors that vary systematically with glacier retreat and advance when applied to observations from Kangerdlussuaq Glacier and Daugaard-Jensen Glacier. The CD model with $d_w = 24$ m performs well at all four sample glaciers (Figure 7c). Indeed, all but one model misfit are less than 2 km, and importantly, misfits appear to be independent of terminus position change for all four glaciers.

Both the VM and SM models with spatially optimized parameter values exhibit model errors that correlate with observed frontal ablation rates. The VM model with $\sigma_{max} = 0.45$ MPa consistently overestimates frontal ablation rates less than 5 m d⁻¹ and underestimates frontal ablation rates larger than 50 m d⁻¹, with one





Figure 8. Comparison of calving model sensitivity and uncertainty across spatial and temporal scales. Position models shown in (a) and rate models shown in (b) with temporal calibrations shown as triangles and spatial calibrations shown as circles. *y* axes display model uncertainty in terms of $m_{75} - m_{50}$ in (a) or $m_{75} - m_{25}$ in (b) associated with spatially optimized model configurations. Uncertainties associated with temporally optimized model configurations are represented by the median uncertainty value of Illullip Sermia, Jakobshavn Isbræ, Kangerdlussuaq Glacier, and Daugaard-Jensen Glacier. Model sensitivity to change in calibration parameter is depicted on *x* axes according to the change in median misfit per "small" parameter change. We define "small" parameter change as 25% of the interquartile range of all observation-optimized parameter values included in our analysis (110 total values).

errors in excess of 100 m d⁻¹ (Figure 7d). Similarly, the SM model with $\sigma_{th} = 0.33$ MPa moderately overestimates observed frontal ablation rates less than 15 m d⁻¹ but substantially underestimates frontal ablation rates that are larger than 50 m d⁻¹ (Figure 7e). Summing misfits for individual glaciers reveals a net underestimation of frontal ablation rates at each glacier by both the VM and SM models.

3.5. Parameter Sensitivities Across Spatial and Temporal Scales

Since we find that observation-optimized parameter values in each calving model are highly variable across both spatial and temporal scales, it is necessary to consider the performance of a calving model not only in terms of its absolute uncertainty, but also in terms of model sensitivity to parameter changes (i.e., to the use of a parameter not ideally optimized to a specific glacier at a specific time). Figure 8 provides a culminating calving model intercomparison achieved by simultaneously comparing measured model uncertainties and model sensitivities across ice sheet spatial scales and seasonal temporal scales.

The CD calving model outperforms the HAF and FAF models across both spatial and temporal scales (Figure 8). When employing spatially optimized and temporally optimized d_w values, the CD model uncertainties in terminus misfit distance are ± 0.27 and ± 0.19 km, respectively. A change in the value of d_w by 25% of its parameter interquartile range results in a change to the CD model bias of around 0.14 km for both spatially optimized and temporally optimized model configurations. Comparatively, the HAF model has uncertainties in terminus misfit distance of ± 0.93 and ± 0.37 km for spatially optimized and temporally optimized h_c values, respectively. This mirrors the finding that a much larger range of observationally optimized h_c values are needed to accurately model terminus position at 50 different outlet glaciers as compared with the range of h_c values that is needed to accurately model terminus positions of an individual glacier over 15 time intervals. That is, there is more variety in the flotation states among different glaciers than there is at a single glacier over time. The median misfits for spatially optimized and temporally optimized HAF model configurations are moderately sensitive to changes in the value of h_c . For change in h_c value equal to 25% of its parameter interquartile range, model bias for the temporally optimized HAF model changes by 0.19 km, while for the spatially optimized HAF model, the model bias changes by 0.16 km. Interestingly, the FAF model performs comparatively worse than the HAF model across spatial and temporal scales, despite similar governing principles. The temporally optimized FAF model demonstrates comparable uncertainty to the HAF and CD models, but exhibits substantially larger model sensitivity to changes in f value.



Our culminating model intercomparison illustrates several important features of the VM and SM calving rate models (Figure 7). Although the VM and SM models feature similar model uncertainties across spatial and temporal scales, the VM model has a significantly higher model sensitivity to changes in free parameter value. Specifically, a change in the value of σ_{max} , equal to 25% of the interquartile range of all observational-optimized σ_{max} values, corresponds to changes of 14 and 22 m d⁻¹ in median rate misfit for temporally optimized and spatially optimized VM model configurations, respectively. For a comparable modification in the value of σ_{th} , both the temporally optimized and spatially optimized SM model configurations exhibit model sensitivities of approximately 3 m d⁻¹. Importantly, the spreads in model uncertainties for both temporally optimized VM and SM model configurations are more than double the uncertainty spreads associated with spatially optimized model configurations. This demonstrates that whereas the VM and SM models can be empirically calibrated to accurately reproduce observed frontal ablation rates at the ice sheet scale for a single time period, both models carry large uncertainty when reproducing short-term frontal ablation rates at individual glaciers over seasonal timescales (Figures 8a and 8b). The reported uncertainties in the spatially optimized VM and SM model configurations calibrated to May/June observations (Table 1) are likely less than the true uncertainties associated with modeled frontal ablation rates during other times of year.

4. Discussion

4.1. Calving Model Performances

The HAF and FAF calving models are able to reproduce observed terminus positions to within roughly 1 km for most outlet glaciers tested in this study. This level of accuracy is adequate for the vast majority of ice sheet models, since at present, only one ice sheet model used in centennial projections runs at a grid resolution of less than 1 km (Goelzer et al., 2018). Although the HAF and FAF models are unable to reproduce floating ice tongues, they are generally not in error by more than 1-2 km at glaciers that terminate in seasonally ungrounded ice. Since the buoyancy heights and buoyancy fractions of individual glaciers investigated here do not change substantially over interannual and seasonal timescales, the uncertainties of temporally optimized HAF and FAF model configurations are much less than spatially optimized model uncertainties. Accordingly, the accuracy of HAF and FAF models will likely persist at grounded glaciers over time, even as glacier geometry and dynamics change. This is an important consideration for prognostic ice sheet models run over centennial timescales with a single, fixed calving criterion. As noted by others (e.g., Benn, Hulton, & Mottram, 2007; Van der Veen, 1996), flotation models fail to account for situations where a glacier terminating in a stable floating ice tongue is transformed into a grounded glacier through disintegration of the ice tongue (e.g., Holland et al., 2008). Since we do not observe this phenomenon in our temporal data set, we cannot quantify the contribution of these relatively rare events to the performance of the HAF and FAF models, though we acknowledge that this phenomenon may be important for those few glaciers that have not yet lost their ice tongues (Hill et al., 2017). Also, because terminal ice thickness is set as a function of fjord geometry in these flotation models, these models would be incapable of bringing about the ice flow accelerations that are often observed at tidewater glaciers as they thin toward the flotation threshold (Enderlin et al., 2018; Kehrl et al., 2017).

Overall, we find that the HAF model slightly outperforms the FAF model in the 50 outlet glaciers selected in this study. We observe that many of the thickest sample glaciers are near flotation while many of the smaller, thinner glaciers are grounded with a greater proportion of ice in excess of flotation. The five thickest sample glaciers are all optimized with critical ice thickness above buoyancy values (h_c) of 12 m or less. Accordingly, a spatially optimized h_c value more accurately accounts for the varying flotation states of sample glaciers than a spatially optimized water depth fraction (f) value. We therefore recommend ice sheet models employing flotation criteria to employ the HAF formula using $h_c = 11$ m.

Of the three position models tested in this study, the CD model reproduces observed terminus positions with the highest degree of accuracy. We find that the CD model is spatially optimized to 50 sample glaciers with 24 m of water in crevasses ($d_w = 24$ m) and has an associated model uncertainty of ±0.27 km. Previous calibrations of the d_w parameter achieved using observations from a single glacier or a numerical ice flow model report optimal d_w values in the range of 0 to 61 m (Choi et al., 2018; Cook et al., 2012; Otero et al., 2010; Otero et al., 2017). Although our observation-optimized d_w values are in the range of



reported values, there is no precedent in the literature for the approach we take to derive a spatially optimized d_w value. We find that spatially optimized CD model exhibits the lowest model sensitivity out of all three spatially optimized position models, though we do find cases on an individual glacier basis where an increase in d_w by 1 m results in substantial terminus position change, similar to results reported in previous studies (Cook et al., 2012; Otero et al., 2017). We confirm previous findings that the CD model can account for ungrounded terminal ice extensions and permanent floating ice tongues (Benn, Hulton, & Mottram, 2007; Todd & Christoffersen, 2014), which is a notable advantage of the CD model over flotation criteria. Additionally, the CD model performed equally well at different times of the year, for all four of our temporally validated glaciers, even without consideration of potential seasonal variations in submarine melt. In the absence of a well-defined means by which submarine melt can be coupled with a calving model, this is a clear advantage.

While we assume that the excellent performance of the CD model indicates that it captures fundamental principles of frontal ablation, namely, a relationship between calving and tensile stress, we acknowledge several inconsistencies associated with the intended physical meaning of d_w as a water depth in crevasses. First, our results indicate a lack of correspondence between optimal d_w values and observed or expected surface crevasse water depths in near terminal regions. We expect water depths in surface crevasses to increase via water input from adjacent ice melt and runoff routing during the early summer months in Greenland, but there is no apparent correlation between calibrated d_w value and time of year in our results, neither do we find a latitudinal gradient in observation optimized d_w values (Figure 3c) that might indicate a dependence on surface mass balance. Additionally, we find instances where optimal d_w values are on the order of tens of meters, but little to no water is observed in surface crevasses near the glacier terminus using 15-m-pixel Landsat 8 satellite images. The lack of correspondence between empirically calibrated d_w values and observed crevasse water depths in this study suggests that the d_w parameter acts as a heuristic stress parameter as opposed to a realistic physical quantity related to the water load in crevasses. We therefore advise against the coupling of the CD model d_w parameter to melt or runoff models.

Furthermore, after a thorough survey of more than 50,000 crevasses from 19 glaciers in Greenland, Enderlin and Bartholomaus (2019) report that the theoretical foundation of the CD model, the Nye formulation of crevasse penetration, has little explanatory power when it comes to crevasse occurrence or depth. Indeed, in zones of compressional stress, the Nye formulation fails to account for the existence of advected crevasses, and in zones of tensional stress, no correlation is found between observed crevasse depths and modeled crevasse depths (Enderlin & Bartholomaus, 2019). These findings support that of previous work from Iceland that found that the Nye model predicted at most 16% of the variance in observed crevasse depths (Mottram & Benn, 2009).

Despite these limitations, the CD model formulation (equations 5 and 7) implies that, at stable termini, ice overburden pressure at the terminus is equal to net horizontal tensile stresses,

$$\rho_i g h = R_{xx} + \sigma', \tag{17}$$

where σ' represents a general stress constant that replaces the downward pressure of water in crevasses, P_w , included in the original CD formulation (Benn, Hulton, & Mottram, 2007) and accounts for any modification to the longitudinal stress balance at the glacier terminus. Our spatially optimized analysis found that terminus position misfit with the CD model is minimized across the Greenland Ice Sheet when $d_w = 24$ m (Table 1), which is equivalent to $\sigma' = 235$ kPa in the case where $\sigma' = \rho_w g d_w$. However, as above, we note that the actual physical mechanism by which the near-terminus, longitudinal stress is increased by 235 kPa remains unidentified.

To explain the demonstrated effectiveness of the CD model in our results here, we consider the possibility that Nye CD errors are balanced by errors associated with overestimation of ice overburden pressure or underestimation of horizontal resistive stresses. These errors could potentially arise from depth-dependent variations in ice viscosity or the overestimation of ice overburden pressure that may occur when the presence of crevasses reduces bulk ice density (Todd & Christoffersen, 2014). Alternatively, surface crevasses may not need to penetrate fully to the waterline or to intersect with basal crevasses to induce calving, as proposed by Bassis and Walker (2012). If this hypothesis is true, overestimated CDs via the Nye formulation may balance



out the error resulting from the overly conservative requirements that $d_s = h$ (or $d_s + d_b = H$) for calving to occur in the CD model (equation 7). A more detailed investigation of near-terminus CDs and stress balance is needed to reconcile the excellent performance of the CD model with the physical inconsistencies of the Nye formulation and the d_w parameter.

The VM and SM calving rate models both reproduce spatially diverse frontal ablation rates from May/June reasonably well. For the VM model, we find a spatially optimized value for the tensile ice strength threshold of $\sigma_{max} = 0.45$ MPa, which is lower than the average of values reported by Choi et al. (2018) that range from 0 to 3 MPa for nine glaciers in the center West and center East of Greenland. Observed frontal ablation rates across the 50 sample glaciers vary by a factor of 0 to 4 times observed ice velocities, which are unaccompanied by corresponding ice stress variations. This renders it challenging to accurately predict all frontal ablation rate observations using the VM model and a fixed value of σ_{max} . The SM model parameter for the threshold stress that must be exceeded to initiate ice damage, σ_{th} , is spatially optimized to 50 sample glaciers with a value of $\sigma_{th} = 0.33$ MPa, which is approximately double the value of $\sigma_{th} = 0.17$ MPa identified via calibration to a diverse set of 13 Arctic marine-terminating glaciers by Mercenier et al. (2018). The SM relation estimates that the ice stresses responsible for calving are a function of ice thickness and water depth. However, we find that seasonal and interannual changes in ice thickness and water depth alone are not large enough for the SM model relation to account for the observed variability in frontal ablation rates. The decrease in observation-optimized parameters for the VM and SM models from temporally distributed observations at Jakobshavn Isbræ and Illullip Sermia implies that near-terminus ice physically weakens during the summer at these glaciers—a phenomenon for which there is no supporting observational or theoretical evidence. Alternately, other important seasonally varying processes (discussed below) must be considered. Both the temporally optimized VM and SM models overestimate low frontal ablation rates observed during nonsummer months while generally underestimating high frontal ablation rates from summer months, which results in calving rate errors of more than 100 m d^{-1} in the most extreme cases (Figure 7). These systematic errors suggest that the VM and SM models are not accurate over monthly or submonthly timescales at glaciers which experience large seasonal fluctuations in frontal ablation rate.

One explanation for the exhibited model deficiencies is that important physical controls on frontal ablation are not accounted for in the current VM and SM calving formulas. Additionally, in the case of the VM model, errors may stem from the assumption that frontal ablation is a linear function of ice velocity (equation 9)— an assumption without theoretical underpinnings. Recent observations and numerical modeling show that ice mélange and submarine melt and undercutting are important drivers of frontal ablation on seasonal timescales, which suppress calving in winter and enhance and potentially amplify calving during the summer runoff season (Cassotto et al., 2015; Fried et al., 2018; Todd & Christoffersen, 2014). Inclusion of these processes in calving rate models may help reconcile overestimates of fall-winter-spring frontal ablation rates and underestimates of summer frontal ablation rate errors of several tens of meters per day would require the most extreme levels of nonlinear dependence put forward by some modeling studies (Ma & Bassis, 2019). Inclusion of some form of submarine melt and its highly sensitive forcing of calving are areas of active research that we leave for further work beyond the scope of this tightly observationally focused study.

Alternatively, we invoke the possibility that the VM and SM models are accurate only over longer timescales in which observed frontal ablation rates are less variable. Indeed, recent works by Choi et al. (2018) and Morlighem et al. (2019) show that an ice flow model using the VM calving model and calibrated to individual glaciers can accurately simulate terminus position change on an annual scale. However, there is evidence that seasonal terminus variability plays an important role in the initiation of major glacier retreat and dynamic destabilization. Observations from West Greenland suggest that a lack of spring advance combined with an extended calving season, potentially as a result of early mélange clearing and warmer fjord water, triggered multiyear retreats at several marine-terminating glaciers in the region in 2003 (Howat et al., 2010). Similarly, the recent retreats of many marine-terminating glaciers in Alaska from stable topographic constrictions began in years when the glaciers did not undergo seasonal advances, also coinciding with above-normal summer sea surface temperatures (McNabb & Hock, 2014). Multiyear retreat also appears imminent at Kangerdlussuaq Glacier in eastern Greenland following two seasons of weak wintertime mélange and expanded calving seasons (Bevan et al., 2019). Ice flow and calving models that do not



account for seasonality in terminus dynamics may therefore miss important dynamic changes that affect long-term discharge projections.

4.2. Calving Rate Models Versus Calving Positions Models

Although our analysis does not permit a direct model intercomparison between calving position models and calving rate models, we consider several lines of evidence that suggest that calving position models are likely more accurate than calving rate models when employed in prognostic ice flow models of the Greenland Ice Sheet. Uncertainties associated with optimized HAF and CD calving models are within the resolution of many Greenland Ice Sheet-scale model runs. Given the correspondence between terminus position change, ice velocity, and ice flux (Howat et al., 2010; Moon et al., 2015), terminus positions modeled by the HAF and CD models that are accurate to within an ice sheet model grid cell therefore minimize ice discharge errors in model projections. Moreover, since we find that errors in the CD model and the HAF and FAF models (when applied to grounded glaciers) do not vary systematically with glacier retreat or advance, we expect model biases for calving position models to remain fairly consistent through time when model parameters are fixed. This conclusion is also supported by the low sensitivities of both spatially optimized and temporally optimized model configurations, as well as the relatively narrow ranges of observation-optimized parameter values needed to account for terminus conditions over time. In comparison, calibrations of optimal parameters in the VM and SM models are not consistent over seasonal timescales, and, for the VM model, parameter values optimized at different glaciers range by a factor of 5 or more (Figure 3). These factors may result in substantial frontal ablation rate errors persisting for at least a portion of every year at individual glaciers or possibly ice sheet wide. Calving rate models therefore have two potentially large sources of model error. The first arises at individual glaciers when the VM and SM models are spatially optimized to yield median bias of 0 m d⁻¹ across the ice sheet as a whole. Although many glaciers have model errors close to 0 m d⁻¹, model errors on the order of 10 m d^{-1} exist at some glaciers. These errors are comparable with measured frontal ablation rates at many glaciers (Figure 5d). The second and likely larger source of model errors stems from the considerable model uncertainties and model sensitivities associated with temporally optimized VM and SM models. Fixed single values for σ_{max} and σ_{th} cannot reliably account for frontal ablation rate variability over time at a single glacier, and especially not at multiple glaciers, without likely generating relatively large model errors at individual glaciers, as is shown in Figures 7d and 7e. Even modest calving rate errors (i.e., biases) that persist at individual glaciers can accrue into potentially large glacier length changes and mass flux errors in prognostic ice flow models when simulations run for decades or more. Given the prospect of accumulated glacier length errors and attendant geometry and dynamic changes, we recommend the use of position calving models over calving rate models for simulation of terminus dynamics in Greenland Ice Sheet models.

Our results for calving position models and calving rate models have important limitations. Using observational data to test calving position models, we are not able to quantify down-fjord terminus misfits. The literature suggests that this is a concern for the CD model, which has been shown to produce unrealistic glacier advance through basal overdeepenings (Amundson, 2016; Nick et al., 2010). However, this erroneous behavior is only found using 1-D flowline models, so it remains unclear whether the incorporation of full horizontal stresses diminishes such unrealistic advances (Todd et al., 2018). Recent testing of the CD calving model in 2-D ice flow models does not reveal unrealistic glacier advance (Choi et al., 2018; Otero et al., 2017), but further evaluation of the CD model using different mass balance forcings and fjord geometries will increase understanding of the advantages and limitations of the CD model when applied to the entire Greenland Ice Sheet. Our focus on observational data also prevented us from including submarine melt in this study and implicitly assuming that all frontal ablation is the result of calving.

The timescale over which frontal ablation processes should be represented also remains poorly understood, a limitation which complicates the formation and evaluation of calving rate parameterizations. Large-scale ice sheet models typically run at time steps of weeks to 1 year (Bindschadler et al., 2013). Our evaluation of VM and SM models against observations from monthly or submonthly timescales therefore only has direct implications for the use of these calving models over corresponding timescales. Our results do not quantify the accuracy of the VM and SM models on annual timescales. Indeed, from our analysis, it is possible that overestimated ablation rates during nonsummer months may partially compensate for underestimated ablation rates during summer months. However, for ice sheet models that resolve outlet glacier dynamics on monthly



or weekly timescales, our analysis provides strong evidence that calving rate models are not able to reliably simulate patterns of observed frontal ablation rates with high fidelity.

4.3. Comparison With Previous Model Validation Efforts and Recommendations for Improved Representation of Calving

Our main findings support and build upon elements of the recent calving model validation work by Choi et al. (2018), Morlighem et al. (2016), and Morlighem et al. (2019), but with important caveats that stem from the different goals of these studies. These prior studies sought to reproduce decadal terminus behavior at individual glaciers (akin to glacier by glacier "temporal optimization") by identifying different optimal parameter values at each study glacier in a manner similar to our "spatial optimization." The results of our observation-optimized calibrations in sections 3.1.1 and 3.1.2 concur with what Choi et al. (2018), Morlighem et al. (2016), and Morlighem et al. (2019) found by fitting calving model parameters to individual glaciers: first, when calving model parameters are optimized to individual glaciers, a wide range of parameter values is needed across different glaciers (0.5 to 2.9 MPa in Choi et al., 2018; 0.4 to 1.2 MPa in Morlighem et al., 2019; and 0.3 to 2.1 for the middle 50% of σ_{max} values found in the present study). This factor of 3 to 5 variation in optimal parameters for the VM calving model exceeds the factor of 3 range of optimal d_w parameters in the CD model. Second, each study, including our own, reports that the performance of the VM model at individual glaciers is highly sensitive to the choice of σ_{max} parameter value. Third, our results also show that the VM model performs very well when optimized to individual glaciers (section 3.1.2), a finding in agreement with the main conclusion of the calving model validations in Choi et al. (2018), Morlighem et al. (2016), and Morlighem et al. (2019).

However, our conclusions and recommendations differ from those of Choi et al. (2018), Morlighem et al. (2016), and Morlighem et al. (2019) due to our more spatially and temporally extensive ensemble calibration approach and our quantification of model sensitivity to parameter change. While focused study of individual glacier change can benefit from glacier-specific parameter tuning, the utility of a calving model that depends upon individually fitted parameter values to reproduce observed terminus dynamics is highly limited since it is undesirable for prognostic ice sheet models to employ unique calving parameter values for each glacier across the Greenland Ice Sheet (e.g., Goelzer et al., 2018). Although glacier-specific parameter tuning generally improves calving model performance at individual glaciers, such calibrations, when applied to a range of sample glaciers that presumably share common physics, lack physical justification and potentially obscures important model shortcomings.

Our ensemble parameter calibrations reveal the necessity of thorough calibration methods when interpreting calving model performances in the context of universal, ice sheet boundary criteria. We find that the accuracy and sensitivity of a calving model that uses a single parameter value for multiple glaciers over multiple years are different from that same calving model using parameter values that are allowed to vary across space and time. For example, when σ_{max} is calibrated to individual glaciers, the VM model can reproduce observed frontal ablation rates at 50 glaciers with zero uncertainty, but when a single σ_{max} value is used for all 50 glaciers, the model uncertainty increases to 6.6 m d⁻¹. Furthermore, when a single σ_{max} value is used to model four glaciers across 8 years, the model uncertainty jumps to 26.4 m d^{-1} (Figure 8). Our analysis also shows that extremely large terminus misfits can arise when calving model parameters are used outside of their calibration range, such as when spatially optimized parameters are used to reproduce time-varying observations as shown in Figure 7. Thus, our results suggest that calving model validations performed at the individual glacier scale do not necessarily inform the utility of that calving model at the ice sheet scale. If the preferred calving model for use in ice sheet simulations employs a single, well-calibrated parameter value for all glaciers and for hundreds of years (as one might prefer for other ice sheet model components, such as a sliding law or constitutive equation), improved calving models should be calibrated across thorough spatial and temporal scales to reflect the universal character of ice sheet boundary criteria. Therefore, owing to our more extensive observational analysis of calving model performance and a different set of study objectives, our results contrast with the calving model recommended by Choi et al. (2018): whereas Choi et al. (2018) recommends use of the VM model tuned to individual glaciers, we recommend the CD for more universal application to the Greenland Ice Sheet as a whole.



Overall, our evaluation of six calving models against spatially and temporally varying observations enables us to recommend the use of calving position models for use as boundary criteria in ice sheet models of the Greenland Ice Sheet. Of the calving position models we tested, the CD criterion with $d_w = 24$ m better reproduced observed terminus positions than the HAF and FAF calving models and, notably, reproduced observed extents of floating ice tongues. In addition to low overall CD model misfit (on the order of 0.2 to 0.3 km), the model misfit was only weakly sensitive to potential parameter changes (0.14 km misfit increase for modest parameter change; Figure 8). However, a lack of correspondence between expected crevasse water depths and calibrated d_w values supports the notion that the CD model should be considered an effective heuristic approach that captures underlying ice stresses and not an exact physical representation of the calving process (Benn, Hulton, & Mottram, 2007). The physical meaning of d_w warrants exploration. Other approaches to predicting terminus position that are formulated in terms of near-terminus ice dynamics, such as a VM yield stress position model (Aschwanden et al., 2019), or accumulated ice damage position model (Krug et al., 2014), should be pursued and evaluated against the CD model.

Lastly, opportunities exist for improved calving rate formulations. Investigation of calving rate model performances over seasonal and interannual timescales reveals the inability of the current VM and SM calving models to account for a wide range of observed frontal ablation rates with fixed model configurations. While the position models perform satisfactorily without consideration of submarine melt processes, the addition of submarine melt and specific undercutting mechanisms that potentially amplify calving may help reconcile the systematic underprediction of summertime frontal ablation rates by increasing near-terminus tensile stresses in calving rate models. Although direct observations of submarine undercutting morphologies will likely remain sparse around Greenland, comprehensive observations of oceanic and fjord conditions by NASA's Oceans Melting Greenland program (e.g., Wood et al., 2018) or through site-specific study (e.g., Fried et al., 2015; Rignot et al., 2015; Wagner et al., 2019) may enable inclusion of realistic submarine undercutting representation in future frontal ablation parameterizations. The coupling of a calving criterion with realistic melt undercutting morphology has recently shown promise in a 3-D full stokes simulation of Store Glacier in West Greenland (Todd et al., 2018). Further observations of frontal ablation rates on varying timescales may constrain the precise physical parameters that control frontal ablation rates on timescales relevant to dynamic glacier changes and that are important to represent in prognostic ice sheet models of Greenland (Fried et al., 2018). To facilitate the goal of further model testing, our data sets are archived at the Arctic Data Center (Amaral et al., 2019).

5. Conclusions

We find that the HAF, FAF, CD, VM yield stress, and SM calving models can be empirically calibrated to individual glaciers, at specific times, to reproduce terminus dynamics at 50 representative glaciers in Greenland with similar accuracy. However, when model free parameters are calibrated to optimal values that best encompass calving observations across the ice sheet and through time, we find varying model performances. The CD model with a resistive stress increase equivalent to 24 m of crevasse-filling water $(d_w = 24 \text{ m})$ reproduces observed terminus positions with high fidelity and outperforms the HAF and FAF models across spatial and temporal scales. All calving position models have greater uncertainty among glaciers than over time, indicating that the performance of ice sheet models utilizing these parameterizations is likely to remain stable even as ice sheet geometry changes.

Optimized VM and SM calving rate models simulate observed frontal ablation rates across 50 sample glaciers reasonably well, but struggle to reproduce seasonal and short-term variability that occurs over many years in observed frontal ablation rates using fixed model configurations. Our results point to the potential for current calving rate model biases to cause substantial glacier length and/or ice flux errors when employed in ice sheet models. Additionally, we find that calving rate models have greater temporal uncertainty than spatial uncertainty. As a result, ice sheet model performance at one time with one ice sheet configurations. Given the calving model formulas tested in this study, we therefore more strongly recommend the use of empirically calibrated calving position models as opposed to calving rate models for use as boundary criteria within ice sheet models of Greenland.



In addition to model strengths, our observational approach to calving model calibration and evaluation reveals important model weaknesses that highlight opportunities to improve representation of frontal ablation processes. Despite the demonstrated accuracy of the CD model on both grounded and floating glaciers, we find inconsistencies in the model's physical representation that point to opportunities for the development of new, stress-based calving position models. Systematic biases exhibited by calibrated calving rate models suggest that additional physical parameters should be incorporated in calving rate models if rate models seek to reproduce observations on monthly timescales. Incorporation of melt undercutting and ice mélange through the coupling of calving rate models to runoff and fjord conditions is likely essential for improved simulation of subannual frontal ablation rates, as has been shown in several process studies (Morlighem et al., 2016; Robel, 2017; Todd et al., 2018; Todd & Christoffersen, 2014). Improved observations of ocean temperatures and surface runoff are necessary to constrain the relationship between melt undercutting and calving in Greenland and to evaluate melt-driven frontal ablation parameterizations that were not tested here (Luckman et al., 2015; Slater et al., 2017).

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